What Difficulties that Students Working in Mathematical Reasoning Questions?

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What Difficulties that Students Working in Mathematical Reasoning Questions?

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Abstract. Reasoning skill plays an important role in developing all other mathematical skills. However, the results of subhan's research (2017) inform that students are still weak and have difficulty in solving mathematical reasoning questions. Therefore, a study to analyze students' difficulties in solving reasoning questions should be conducted to overcome this problem. Participants in the study were 15 of the 25 public high school students in Bantul district of Yogyakarta who had completed mathematical reasoning questions. They were deliberately chosen because when they completed mathematical reasoning questions from 25 students only 15 students completed mathematical reasoning questions in their entirety. The instruments of mathematical reasoning are as many as five questions in the class XI line material that has been tested for quality.The analysis was performed using types of difficulties adapted from procedure Newman which includes (1) reading, (2) comprehension, (3) transformation, (4) process skill, and (5) encoding. The results of the study indicate to encoding difficulties of 37.93%, comprehension difficulties of 24.14%, transformation difficulties of 19.54%, and process skill difficulties of 18.39%.

1. Introduction

Reasoning is an important ability to train students when learning mathematics. The importance of reasoning ability is evidenced by the inclusion of reasoning in mathematical learning objectives. More broadly, the importance of mathematical reasoning ability was also proven that the National Council of Teachers of Mathematics included reasoning abilities as one of the five standard processes in mathematics learning. Habsah [1] explains that reasoning skills are very important in mathematics learning. This importance is so that Thompson [2] likens reasoning to the pulse of mathematics. Sumarmo in more detail [3] reveals that mathematical reasoning abilities are very important in mathematical understanding, exploring ideas, estimating solutions, and applying mathematical expressions in relevant mathematical contexts, and understanding that mathematics is meaningful. The importance of reasoning in mathematical understanding and exploring ideas shows that reasoning emphasizes mindset. Santrock [4] which explains that reasoning is a logical mindset using induction or deduction to reach conclusions. Lithner [5] explains that reasoning is a mind pattern to produce a statement and conclusion in solving problems. Based on the narrative of the experts above it can be

concluded that reasoning is a very important ability in mathematics and emphasizes the logical mind pattern to obtain conclusions.

Logical conclusions in reasoning is one that has very complex implications for students because students must process the mind to produce the right conclusions. This complexity often causes reasoning including abilities that are not easily achieved by students [6]. The scores of the ability of overall high school students or grouped based on the cognitive stages of students in solving mathematical reasoning questions were still low [7]. The study by Priata [8] regarding mathematical reasoning also suggested that the quality of students' mathematical reasoning skill was low, which was 49% of the ideal score. Other studies also showed that students had low mathematical reasoning skill [9].

Other facts show that the importance of mathematical reasoning is not acknowledged with real efforts to improve mathematical reasoning skill of the students. Widdiarto [10] argued that in the learning process, teachers rarely emphasize the process that enables students have a practice on reasoning skill and still use less varied methods to deliver the materials. This results in students' tendency to memorize and think mechanistically which signify low level abilities. In fact, high-order thinking skill should be taught to students at the secondary school level. In reality, the high-order thinking skills of the students are still low. The study by Delima [11] indicated that 6% of students had low reasoning skill. Although students who lack reasoning skill are low in numbers, it should become a concern for researchers to provide solutions so that students are able to increase mathematical reasoning skill.

The low level of mathematical reasoning abilities of students conveyed by various sources above indicate the difficulty of students in completing mathematical reasoning questions. Difficulties in solving mathematical reasoning problems if left unchecked will have a negative impact on students because reasoning is the foundation in learning mathematics [12]. Therefore, early analysis is needed regarding any difficulties experienced by students in solving mathematical problems. By doing so, it is expected that the results of studies can be useful for teachers or other related parties so that they are able to provide proper solutions to improve students' mathematical reasoning skill.

2. Methods

The study aims to identify, describe, and clarify students' difficulties in solving mathematical reasoning questions. Regarding triangulation, researchers used interviews with students as clarification and reinforcement of the findings. In the current study, the researchers identified and clarified what difficulties students faced in solving mathematical reasoning questions. Furthermore, types of difficulties were classified using Newman's Error Procedure (NEP) that consists of reading error, comprehension error, transformation error, process skill error, and encoding error. The participants studied were 15 high school students in grade XI of science program in Bantul Sub-District. Students came from heterogeneous academic skills. Data collected using a test consisting of five items of mathematical reasoning in the form of a description of the sequence material. The five items of reasoning were adopted from the development of mathematical reasoning questions. The items used in this study have been tested for quality through the process of content validation and construct validation.

Data were analyzed qualitatively using Miles & Huberman's model and quantitatively to calculate the percentages of students' difficulties. Miles & Huberman's model consists of three phases which are data reduction, data display, and conclusion drawing. In the data reduction phase, the researchers examined every process of solving mathematical reasoning questions and classified it into correct and incorrect process. Afterwards, in the data display phase, the researchers classified the errors students made based on Newman's Error Procedure. This procedure suggests 5 (five) specific stages that serve as an important guidance to discover where the errors occurred in students' works when solving essay questions, which are: (1) reading, (2) comprehension, (3) transformation, (4) process skill, and (5) encoding. Students' difficulties discovered in this study are described in details in the following table.

Table 1. Description of Difficulties Adapted from Newman

No	Types of Difficulties	Indicators of Difficulties
1	Reading	• Unable to recognize words written in the question • Unable to interpret difficult words in the question
2	Comprehension	• Unable to comprehend certain sentences in the question • Unable to correctly write what is required in the question
3	Transformation	• Transform the information in the question into mathematical words incorrectly • Unable to choose formula or theory correctly
4	Process Skill	• Unable to compute easily • Unable to explain the process of computation correctly in the answer sheet • Unable to continue the solving procedure
5	Encoding	• Unable to interpret the answer correctly (Unable to transform to the beginning form correctly)

Students' errors were classified in accordance with Newman's Error Procedure. Quantitative analysis was performed to calculate the percentage of each type of errors. In each question, a student made more than one error. For example, for the question number 1, a student faced difficulties in comprehension and transformation. To find out the percentage of each type of errors in NEP, the number of students who made each error was divided by the total number of participants. In the conclusion drawing phase, the researchers made conclusions regarding the analysis on students' difficulties in each type of errors.

3. Results and Discussion
3.1. Results
The analysis in the reduction phase informs that the researcher checks the answers of 15 students based on the completion steps in each item. The researcher classified the process of solving students in each item as a wrong and correct process. The results of the analysis inform that there are 87 wrong resolution processes where things indicate student difficulties. Furthermore, in the data presentation phase, 87 students 'difficulties in solving students' mathematical reasoning problems are grouped according to Newman's difficulty procedure. The following is a recapitulation of the percentage of students' difficulties in solving mathematical reasoning questions in each item.

Table 2. Percentages of Students' Difficulties

Question No	Percentages of Difficulties					
	Reading	Comprehension	Transformation	Process Skill	Encoding	Total
1	0(0%)	4 (4.60%)	2 (2.30%)	4 (4.60%)	7 (8.05%)	13(19.54%)
2	0(0%)	6 (6.90%)	5 (5.75%)	2 (2.30%)	6 (6.90%)	19(21.84%)
3	0(0%)	4 (4.60%)	3 (3.45%)	0 (0.00%)	2 (2.30%)	9(10.34%)
4	0(0%)	5 (5.75%)	3 (3.45%)	6 (6.90%)	8 (9.20%)	25(26.29%)
5	0(0%)	1 (2.30%)	4 (4.60%)	4 (4.60%)	9 (11.49%)	20(22.99%)
Total	0(0%)	21(24.14%)	17(19.54%)	16(18.39%)	33(37.93%)	87(100%)

Table 2 illustrates that in general most students experienced difficulties when doing the question number 4 with a percentage of 26.29%. On the other hand, students encountered the least difficulties when doing the question number 3. Regarding each type of students' difficulties, it can be seen that in solving mathematical reasoning questions, students were more likely to experience difficulties in encoding with a percentage of 37.93%, followed by difficulties in comprehension with a percentage of 24.14%, in transformation with a percentage of 19.54%, and process skill with a percentage 18.39%. In the last phase of conclusions, the results of the analysis show that the difficulties of students in

completing the biggest mathematical reasoning questions in a row, namely in encording, comprehension, transformation, process skills. Furthermore, analysis of student difficulties in each type of difficulty performed in working on mathematical reasoning questions will be presented in the following section.

3.1.1   Difficulties in Comprehension .Difficulties students faced in comprehending the questions are indicated by the process of solving mathematical reasoning questions which includes if students wrote information or what they knew from the question correctly and understood certain sentences in the question. Based on the analysis results presented in Table 1, it can be seen that the percentage of difficulties in this type of error is 24.14%. The following is an example of students' difficulties in comprehending the question number 2.

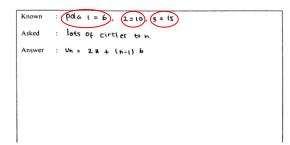Figure 1. Students' difficulties in comprehending the question number 2

Figure 2. Students' difficulties in comprehending the question number 1

The answer students gave to solve the question number 2 indicates that students found it difficult to comprehend the question. Students did not write the information required in the question completely and transform information correctly. In the question number 2, the two terms have a constant ratio. Ratio means that the sequence is geometric sequence so that students were supposed to find out the ratio and the first term to continue the sequence. In reality, however, students solved the question number 2 by misunderstanding it as an arithmetic sequence. Based on the results of interview, students said that the word "ratio" written in the question was understood as a characteristic of an arithmetic sequence; thus, they used a formula of arithmetic sequence. This finding indicates that students failed to understand the question. This failure is more likely to be caused by the fact that students lacked the ability to comprehend the question well. Therefore, students encountered difficulties to understand the problem given, were not able to write the important information in the question, and failed to understand the requirement of the question.

In Figure 2, it can be seen that students wrote the information contained in the question number 1 by using symbols they made themselves without clear explanation. For example, a student wrote "the pattern 1=6, 2=10, and 3=15". Besides, the information students wrote was not in accordance with the context of the question. This problem is denoted when students wrote the number of pattern 1, pattern 2, and pattern 3 based on the black and white circles. However, students were supposed to write the information regarding the number of pattern 1, pattern 2, and pattern 3 based on the black circles only. The error students made in writing the information in the question number 1 shows that students experienced difficulties in comprehending questions so that they failed to investigate the patterns. The results of interview indicate that students made this error because they thought that the symbols they wrote as the answer already gave information in accordance with the question. Although the symbols the students used were different, they thought that people were still able to understand the answer they wrote. Furthermore, the error in investigating the pattern was caused by the fact that students did not focus their attention on the process of understanding the question. Students were more likely to focus by looking at the pattern presented in the picture without reading the question accurately.

3.1.2 Difficulties in Transformation. Difficulties in transformation occurred when students correctly comprehended what the question required, but were not able to identify the correct mathematical operation or sequence of operation to successfully solve the question. Based on the results of analysis in Table 1, in general the total percentage of this type of difficulties is 19.54%. An example of transformation difficulties is presented in the following figure.

Figure 3. Transformation difficultie in the question number 4

Based on Figure 3, students transformed information in the question number 4 into a correct mathematical sentence. However, one part was not translated correctly. It is shown in the question that the price of the seats in the first row was the most expensive and denoted with variable x. Students wrote the price of the seats in the first row was x and in the next row was x+10,000, and etc. The notation x+10,000 shows that the seats in the second row were more expensive than in the first row. However, this is not in line with the information in the question that x was the most expensive price of seats. Therefore, students' error in this question indicates difficulties in transformation. The results of interview suggest that students understood that the most expensive ticket was x, but due to the incorrect concept, they added 10,000 to the next row.

3.1.2   Difficulties in Process Skill. Difficulties in process skill are shown by students' error in doing mathematical operations, inability to do sequence of operations, or inability to explain the process of operations. Based on the results of analysis in displayed in Table 1, the difficulties in process skill are 18.39%. The following is an example of students' error in process skill for the question number 1.

Figure 4. Difficulties in process skill in the question number 1

Figure 5. Difficulties in process skill in the question number 4

Figure 4 shows that students make mistakes in the calculation process looking for the number of black spheres in the nth term so that students cannot continue the calculation process performed. In addition, other findings indicate that students do not understand the problem so they do not explain the calculation process. The error was shown when making the operation of "(n-1).2". Students wrote the results of operation of (n-1).2 as 2n-1, although the correct operation was supposed to be 2n-2.

Students' error in this part suggests that students experienced difficulties in process skill of calculating integers. The results of interview indicate that students were confused when doing the calculation in the form of (n-1)2. This form is one of the properties of operations with integers called commutative. When students were given questions related to properties of operations with integers, they were not able to solve them accurately. Therefore, students faced difficulties in process skill due to low comprehension and process skill in calculation.

Figure 5 shows that students did not explain their calculation process when solving the question number 4. After looking for the value of 'a', students wrote one row of numbers and did not give information. No information suggests that students faced difficulties in process skill. The results of interview demonstrate that students made an error in this part because it was difficult for them to make a correct procedure. They had difficulties finding what parts to look for to obtain the right answer.

3.1.3   Difficulties in Encoding. Difficulties in encoding occurred when, despite being able to solve the question, students failed to interpret the answer in accordance with the context, wrote incorrect answers, or even did not write the answers at all. Based on the results of analysis in presented in Table 1, the percentage of difficulties in encoding is 37.93%. The following is an example of students' answers when solving the question number 5 and number 1.

Figure 6. Difficulties in encoding for the question number 5

Figure 7. Difficulties in encoding for the question number 1

Students could calculate correctly to solve the question number 5. However, Figure 6 shows that the conclusion students drew was not in accordance with the question. The question number 5 asks which bank Budi has to choose in order to obtain profit complete with the reason. Students, however, wrote that the bank that gave the most profit was BCI. Students were supposed to write "Budi has to choose BMI because it gives the most profit as much as 2,400,000 rupiahs." Based on the results of interview with students, students wrote that way because they thought that their answer was already able to solve the question. Thus, in conclusion drawing, students failed to write the proper and acceptable form of answer. In other words, students were not able to interpret the answer to be in accordance with the question's requirement.

The question number 1 asks how many black circles are in the nth term. Based on Figure 7, it can be seen that students successfully found the number of black circles in the nth term. However, when drawing a conclusion of the answer to the question number 1, the majority of students wrote "the formula for the n term is 2n+3." This is not a proper conclusion because, despite 2n+3 being the correct formula, students were not able to provide the form of answer in accordance with the context. The conclusion students were supposed to write is "the number of black circles in the nth term is 2n+3". Based on the results of interview, it is discovered that students successfully understood the procedure to solve the question number 1, but when giving the final answer, they thought their answer already solved the question without relating it to the context of the question. Students often use this

concept; they successfully execute the mathematical processes in order to solve the question, but they often fail to interpret the results of their calculation used as the answer.

3.2. Discussion

Results of the study show that students experienced difficulties in solving mathematical reasoning questions. The errors they made signify the difficulties. The analysis of students' difficulties indicates that when attempting to solve the mathematical reasoning questions, students faced difficulties in comprehension, transformation, process skill, and encoding. This is in line with the results of studies by Prakitipong, et al, [13] and Layn [14], showing that the difficulties students encountered in solving PISA questions included difficulties in comprehension, transformation, process skill, and encoding. Studies by Trapsilo [15] and Dewi, et al, [16] also yield the same result that no students had no difficulties in reading mathematical reasoning questions. This finding signifies that the research participants who were already high school students psychologically had a good reading skill. Besides, the questions tested on students had good quality in terms of content validity so that students were able to understand the information contained in the questions easily.

In solving reasoning questions, the most difficulties faced by students were encoding and comprehension. The difficulties in encoding in this research occurred when students wrote the conclusion or the final result of the calculation. This happened because students did not interpret the answer in accordance with the context of the questions and were more likely to write the answers only in the form of numbers. This is in parallel with the finding of the study by Cahyaningrum [17], showing that students often failed to write the conclusions according to the context in solving mathematical reasoning questions. The difficulties in this process are often faced by students in solving mathematical questions or any problems in daily life. The biggest factors are students are more likely to do mathematical questions in a rush and to think that it should suffice to write the numbers as the answer to the question without adding words to relate it with the context of the question. Rahman and Fauziana [18] argued that such cases can be frequently found in the field. However, these frequent occurrences influence students' learning outcomes and achievements.

This study found that comprehending the mathematical reasoning questions became one of the difficulties most students faced. It was difficult for students to understand the questions or to construct procedures to solve the questions. The study conducted by Phonapichat, et al, [19] found similar result which students did not have patience and like reading the questions. Karimah and Fuad [20] suggested that in solving PISA questions to test reasoning skill, students experienced difficulties in the attempt to comprehend the questions. Students did not write the complete information required in the questions and failed to interpret certain words or sentences. The ability to comprehend questions is a vital aspect of literacy. The results of study by PISA in 2015 [21] signified that mathematical literacy that includes the ability to comprehend mathematical tasks is low. Based on this finding, 20.3% of Indonesian students were below level 1 (the score was below 334.94). The study also reported that Indonesian students were able to comprehend only 30% of the reading material because solving questions that needed comprehension and reasoning skills was difficult for them [22].

Difficulties in transformation were also experienced by students in their attempt to solve mathematical reasoning questions. They failed to use the correct mathematical formulas, to transform information in the questions into mathematical words, and to perform procedures consistent with the questions' requirements. These findings suggest a similarity with the results of the study by Samsul, et al, [23], indicating that in doing transformation, students made errors in determining the formula to solve the mathematical tasks and this has become evidence that not all students have a good understanding of concepts. Furthermore, White [24] argued that regarding difficulties in transformation, students already comprehend the information and the requirement of the questions, but they were not successfully able to identify the operation or sequence of operations necessary to solve the questions. Wijaya, et al, [28], in his study, the lowest difficulty is in process skills. Difficulties in process skills tend to be context-based and therefore do not reach the stage of bringing mathematical procedures out. This can be interpreted that even though students experience difficulties in process

skills such as miscalculations or not completing the results of calculations, the resolution process does not deviate from mathematical procedures.

4. Conclusion

Difficulties faced by students in solving mathematical reasoning questions comprise difficulties in comprehension, transformation, process skill, and encoding. Based on quantitative analysis, encoding was the most difficult stage of difficulties with 37.93%, followed by comprehension with 24.14%, transformation with 19.54%, and process skill with 18.39%. Students' difficulties in attempting mathematical reasoning questions were caused by a number of factors. First, students found comprehending the questions difficult. Second, students used words or symbols that are not commonly used so that people did not understand what students wrote and could not focus their attention in comprehending the questions. Third, students' ability to perform mathematical operations and to interpret the questions in accordance with the context was still low. Students' difficulties found in this study suggest that efforts need to be made so that students' abilities to solve mathematical reasoning questions can be improved. Therefore, this study can serve as a start for the next research.

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